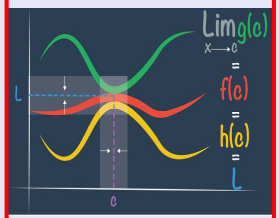
Calculus I

Lecture 36



$f(x) = x^4$

Polynomial $\rightarrow$ Cont. & diff. $(-\infty, \infty)$

Domain $(-\infty, \infty)$ No V.A.

Range $\rightarrow f(x) = x^4 \geq 0 \rightarrow$ Range $[0, \infty)$

x-Int & y-Int $(0,0)$

Repeated 4 times $\rightarrow$ even # of times (You only touch x-Int)

$f'(x) = 4x^3$ C.P. $\rightarrow f'(x) = 0 \rightarrow x = 0$

$(0,0)$

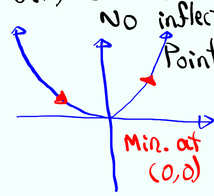
$f''(x) = 12x^2$ P.I.P. $\rightarrow f''(x) = 0 \rightarrow x = 0$

$(0,0)$

$f''(x) \geq 0 \rightarrow f(x)$ is C.U.

No inflection Point

x	$-\infty$	0	∞
$f'(x)$	$-$	•	$+$
$f''(x)$	$+$	•	$+$
$f(x)$	↘	Min.	↗



$f(x) = \dfrac{x^3}{x+8}$

Domain $x \neq -8$ → V.A. $x=-8$

Y-Int $(0,0)$ x-Int $f(x)=0$ $\dfrac{x^3}{x+8}=0$ $x^3=0$ $\boxed{x=0}$ odd # of times

If $\dfrac{x}{x+8}$ → H.A. $y=1$

If $\dfrac{x^2}{x+8}$ → slant Asymptote

$f(x) = \dfrac{x^3}{x+8}$ → Quadratic

Long Division

$$\begin{array}{r|l} & x^2 \quad -8x \quad +64 \\ \hline x+8 & x^3 + 0x^2 + 0x + 0 \\ & -(x^3 + 8x^2) \\ & \quad -8x^2 + 0x + 0 \\ & \quad -(-8x^2 - 64x) \\ & \qquad 64x + 0 \\ & \qquad -(64x + 512) \\ & \qquad\quad -512 \end{array}$$

$f(x) = x^2 - 8x + 64 + \dfrac{-512}{x+8}$

$f'(x) = 2x - 8 + \dfrac{512}{(x+8)^2}$

$f''(x) = 2 - \dfrac{1024}{(x+8)^3}$ ← Make the Correction.

C.P. $f'(x)=0$ or $f'(x)$ is undefined

P.I.P. $f''(x)$ or $f''(x)$ is undefined

find all C.P., and P.I.P.

Do sign chart, then do the graph
(finish it over weekend)

Oct 31-7:49 AM

$f(x) = \dfrac{x^2}{x+8}$

Domain $(-\infty,-8)\cup(-8,\infty)$ V.A $x=-8$

Y-Int $(0,0)$

x-Int $(0,0)$ Repeated twice (even # of times)

$$\begin{array}{r|l} & x \quad -8 \\ \hline x+8 & x^2 + 0x + 0 \\ & -(x^2 + 8x) \\ & \quad -8x + 0 \\ & \quad -(-8x - 64) \\ & \qquad 64 \end{array}$$

$f(x) = \underbrace{x-8}_{\text{slant Asymptote}} + \dfrac{64}{x+8} \to 0$ as $x \to \infty$

slant Asymptote $y = x-8$

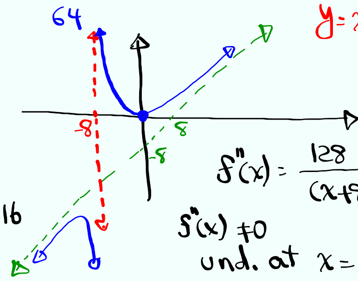
$f'(x) = 1 - \dfrac{64}{(x+8)^2}$

$f'(x)=0$ $1 - \dfrac{64}{(x+8)^2} = 0$

$(x+8)^2 = 64$ $x+8 = \pm 8$

$x=0$, $x=-16$

$f'(x)$ is undefined at $x=-8$



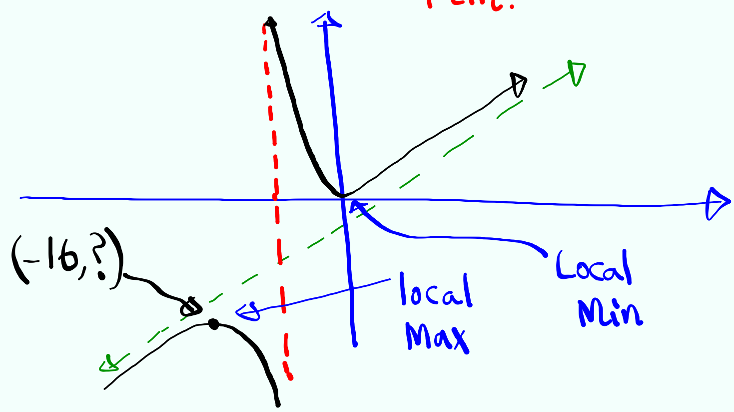
$f''(x) = \dfrac{128}{(x+8)^3}$

$f''(x) \neq 0$

und. at $x=-8$

Nov 4-7:35 AM

x	$-\infty$		-16		-8		0		∞
$f'(x)$		$+$	0	$-$	0	$-$	0	$+$	
$f''(x)$		$-$		$-$	0	$+$		$+$	
$f(x)$		↗	Max	↘		↘	Min.	↗	



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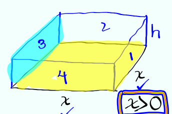
A box has a square base & open top.



Volume is 32000 cm^3

$x \cdot x \cdot h = 32000$

$\boxed{x^2h = 32000}$

$\boxed{x>0}$

Find dimensions with least amount of materials to make the box.

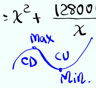
$\underbrace{x^2}_{\text{base}} + \underbrace{4xh}_{\text{sides}}$ Minimize

$x^2 + 4x \cdot \frac{32000}{x^2}$

$f'(x)$ & $f''(x)$

Determine concavity & critical point

$f(x) = x^2 + \frac{128000}{x}$



$f'(x) = 2x - \frac{128000}{x^2}$

$f''(x) = 2 + \frac{256000}{x^3}$

$f'(x) = 0$

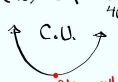
$2x^3 - 128000 = 0$

$x^3 = 64000$

$x = \sqrt[3]{64000}$

$\boxed{x = 40}$

$f''(40) = 2 + \frac{256000}{40^3} > 0$

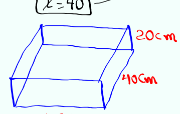


Min. when $x=40$

$x^2h = 32000$

$40^2h = 32000$

$\boxed{h = 20}$



How much materials needed?

$x^2 + 4xh$

$40^2 + 4(40)(20)$

$= \boxed{}\ cm^2$

Oct 31-8:10 AM

Verify the conditions for Rolle's thrm for
$f(x) = x^3 - x^2 - 6x + 2$ over $[0,3]$
then find all values ove $[0,3]$ for
the conclusion of Rolle's thrm.

$f(x)$ cont. $[a,b]$, $f(x)$ diff. (a,b), $f(a)=f(b)$

$f'(c)=0$ where $c \in (a,b)$

$f(x)$ is polynomial $\Rightarrow$ Cont. & diff. $(-\infty,\infty)$

$f(0) = 0^3 - 0^2 - 6(0) + 2 = 2$ ✓

$f(3) = 3^3 - 3^2 - 6(3) + 2 = 27 - 9 - 18 + 2 = 2$ ✓

$f'(x) = 3x^2 - 2x - 6$

$f'(x) = 0 \qquad 3x^2 - 2x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{4+72}}{6}$$

$$x = \frac{2 \pm \sqrt{76}}{6} \qquad \boxed{x \approx 1.786 \rightarrow (0,3)}$$

$x \approx -1.120$ not $(0,3)$

Nov 4-8:04 AM

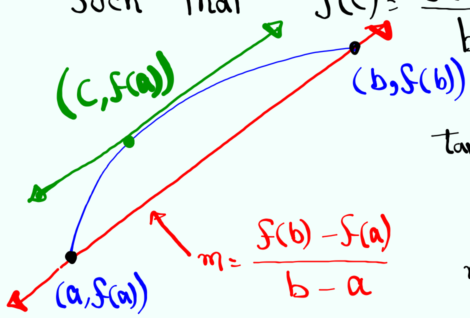
Mean - Value Thrm

$f(x)$ is cont. on $[a,b]$

$f(x)$ is diff. on (a,b)

then there is at least a number c in (a,b)

Such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.



tan. line ∥ Secant line

Same slope

$m_{\text{tan. line}} = m_{\text{Sec. line}}$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Nov 4-8:12 AM

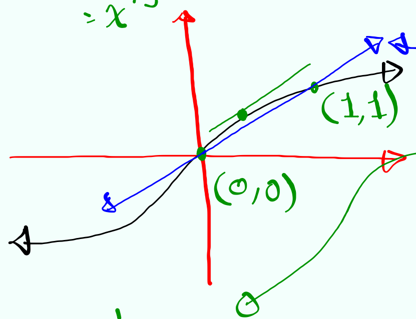
$f(x) = \sqrt[3]{x}$

$= x^{1/3}$

$[0,1]$

$f(x)$ is cont $[0,1]$

$f(x)$ is diff $(0,1)$



Secant line

$$m = \frac{1-0}{1-0} = 1$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

$$f'(x) = 1$$

$$\frac{1}{3\sqrt[3]{x^2}} = 1 \qquad \sqrt[3]{x^2} = \frac{1}{3}$$

$$x^2 = \frac{1}{27}$$

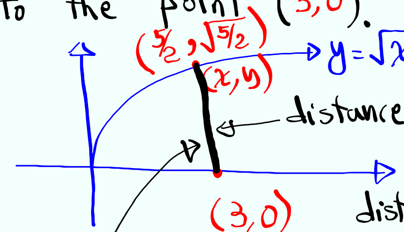
$$x = \sqrt{1/27}$$

$$\boxed{x \approx .192}$$

is in $(0,1)$

Nov 4-8:20 AM

Find a point on $y=\sqrt{x}$ that is closest to the point $(3,0)$.



$y=\sqrt{x} \rightarrow y^2 = x$

distance to be minimum

distance formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = \sqrt{(x-3)^2+(y-0)^2}$$

$$d = \sqrt{(x-3)^2+y^2}$$

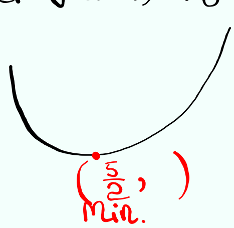
$$d = \sqrt{(x-3)^2+x}$$

$$f(x) = (x-3)^2+x$$

$$f'(x) = 2(x-3)^1+1$$

$$f'(x) = 2x-5$$

$$f''(x) = 2 > 0$$



Nov 4-8:26 AM

$f'(x) = 1 + 2x$

$f(2) = 5$

find $f(x)$.

$f(x) = x + x^2 + C$

General Solution

$f(2) = 2 + 2^2 + C = 5$

$6 + C = 5$

$\boxed{C = -1}$

$\boxed{f(x) = x + x^2 - 1}$

Specific Solution.

Nov 4-8:33 AM